LBNL-41447
Preprint

A Comparison of the Power Law to Quadratic Formulations for Air Infiltration Calculations

Iain S. Walker, David J. Wilson,
and Max H. Sherman

Environmental Energy Technologies Division

March 1997

Submitted to
Energy and Buildings

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, or The Regents of the University of California.

Ernest Orlando Lawrence Berkeley National Laboratory
is an equal opportunity employer.

LBNL-41447

A Comparison of the Power Law to Quadratic Formulations for Air Infiltration Calculations

Iain S. Walker, David J. Wilson and Max H. Sherman

Environmental Energy Technologies Division
Indoor Environment Department
Lawrence Berkeley National Laboratory
Berkeley, CA 94720

University of Alberta
Department of Mechanical Engineering
Edmonton, AB, Canada

March 1997

This work was supported by the Assistant Secretary for Energy Efficiency and Renewable Energy, Office of Building Technology, State and Community Programs under U.S. Department of Energy Contract No. DE-AC03-76SF00098.

A comparison of the power law to quadratic formulations for air infiltration calculations

Published in Energy and Buildings, Vol. 27, No.3, June 1997.

Iain S. Walker*, David J. Wilson and Max H. Sherman***

* Energy Performance of Buildings Group
Energy and Environment Division
Lawrence Berkeley National Laboratory
Berkeley, CA, USA

** Department of Mechanical Engineering
University of Alberta
Edmonton, AB, Canada

1 Synopsis

Although the power law has been broadly accepted in measurement and air infiltration standards, and in many air infiltration calculation methods, the assumption that the power law is true over the range of pressures that a building envelope experiences has not been well documented. In this paper, we examine the validity of the power law through theoretical analysis, laboratory measurements of crack flow and detailed field tests of building envelopes. The results of the theoretical considerations, and field and laboratory measurements indicate that the power law is valid for low pressure building envelope leakage.

2 Introduction

The functional form of the pressure flow relationship for building envelopes has been a topic of debate. Historically, some practitioners supported a power law equation [1] and others a quadratic form [2]. The power law formulation has gained almost universal acceptance for building envelope leakage in:

- measurement standards for building envelopes, e.g., [3], [4], [5],
- ventilation standards, e.g., [6] and [7], and
- many infiltration models.

Many of these standards and calculation procedures use the power law function to extrapolate from data measured at high pressure differences down to the pressures experienced by the building envelope for natural infiltration. This paper will examine how well the power law and quadratic functions can be extrapolated successfully to lower pressures by using theoretical considerations, laboratory and field measurements. In addition, this paper examines how flow through individual leaks combine when determining whole building envelope flows. Test results will be presented for whole house pressurization at the low temperature differences and windspeeds required to reveal the low pressure leakage function. Additional crack flow measurements performed by other authors and flow through furnace flues under controlled laboratory conditions will also be used. The envelope and flue experiments were developed to concentrate on improved measurements at low pressure differences and flow rates.

$$A = \frac{128\mu z}{\pi D^4}, \quad B = \frac{\rho Y}{2d^2L^2} \qquad \textbf{7, 8}$$

where D is the pipe diameter, and m is a factor to account for the linearization of the Navier-Stokes equations.

The quadratic equation allows the flow to vary from laminar to turbulent over a range of flow rates. However, this equation is based on combining fully developed laminar and turbulent flows and entry and exit losses. This can be physically unrealistic for the convoluted crack geometries typical of building leaks in which the flow is rarely fully developed because the flow has to begin its development after each sharp change of direction. In addition, the pressures across building leaks are not steady because of wind turbulence. This results in changing driving pressures for the flow such that the flow is being accelerated or decelerated almost all of the time. The fluctuations in flow and pressure further reduce the possibility of fully developed flows existing in building leaks.

3.2 Power law form

The power law relationship has the form

$$Q = C\Delta P^n \qquad \textbf{9}$$

where C [m^3/sPa^n] is the flow coefficient and n is the flow exponent. The flow exponent has the limiting values of 0.5 and 1 for fully developed turbulent and laminar flows respectively. A dimensionless pressure has been developed [12] that relates the ratio of total pressure drop to the critical pressure drop that occurs when the pressure drop due to fully developed laminar flow is equal to the pressure drop from combined entry, exit and flow acceleration effects. This parameter, S, has been related to the power law exponent, n , which allows the power law exponent to be related to the crack geometry, such that

$$S = \frac{1}{8}\frac{(1-n)n}{\left(n-\frac{1}{2}\right)^2} \qquad \textbf{10}$$

where A is the cross sectional area of the crack. The flow can then be expressed as a function of S:

$$Q = \frac{16\pi\nu z}{m}\phi S^n \qquad \textbf{11}$$

where ν is the kinematic viscosity and ϕ is a power law factor depending on the exponent, n. Temperature and pressure corrections for the flow coefficient, C, can be made as follows (some of which was suggested previously [16]). From dimensional analysis it can be shown that

$$C \propto \frac{\rho^{n-1}}{\mu^{2n-1}} \qquad \textbf{12}$$

Given typical building crack geometries and flow rates the flow in building leaks is likely to be developing flow. Some researchers suggest that the flow exponent, n, is constant over a wide range of flow rates and pressure differences for cracks similar in geometry to building leaks. For example, for laminar flow in the entrance region of smooth circular tubes [17]. It has been proposed [18] that the results in [17] imply an exponent of n = 2/3 for this entrance region developing flow regime. This is also a typical value for n found from pressurization testing of houses. Although tempting, this does not prove that flow in cracks in building envelopes is undeveloped laminar flow because the developing flow regime in [17] was only dominant over an entry length of less than one diameter. It remains an intriguing coincidence, however, and requires further research. Experiments on parallel flat plates [19] have shown that n is constant over a very wide range of flow rates and pressures for a given crack geometry. The tests were performed from 1 to 50 Pa, encompassing the typical values experienced by a building envelope.

Other work has found that the power law exponent, n, may vary with flow rate. Tests of circular capillary tubes with length to diameter (aspect) ratios ranging from 0.45 to 17.25 found that n depends on aspect ratio for laminar flow where $Re_D < 2000$ (Re_D is Reynolds number based on tube diameter, D) [20]. Most building leakage sites fall into this category. For example, a 1 mm diameter crack with orifice type flow will have a $Re_D \approx 85$ for 1 Pa pressure drop and $Re_D \approx 400$ for 10 Pa pressure difference. The capillary tube measurements showed that at high aspect ratios the flow became more laminar and n approached 1, while at low aspect ratios the entrance effects were more dominant and n approached 1/2.

5 Flow through arrays of cracks

Previous work [11], [19] and several other researchers has concentrated on flow through an individual crack or cracks in series. However, in a real building the total leakage is the sum of many individual cracks of differing flow characteristics in series and parallel with each other that are distributed over the building envelope.

5.1 Parallel Cracks

The flow may be modeled as a parallel array of cracks. For laminar flow

$$\Delta P_L = R_L Q_L \qquad \textbf{17}$$

where ΔP_L is the pressure drop across the laminar flow crack, R_L is the flow resistance and Q_L is the flowrate. Similarly, for orifice like cracks

$$\Delta P_O = R_O^2 Q_O^2 \qquad \textbf{18}$$

where ΔP_O is the pressure drop across the orifice flow crack, R_O is the flow resistance and Q_O is the flowrate. For cracks in parallel an electrical analogy is to have the flow resistances in parallel such that

$$Q_{total} = Q_L + Q_O \text{ and } \Delta P_{total} = \Delta P_L = \Delta P_O \qquad \textbf{19, 20}$$

Substituting Equations 17 and 18 in Equation 19 and using Equation 20 gives

testing. The buildings were tested with the large holes (e.g., furnace flues) sealed to observe pressurization test results for arrays of parallel and series cracks. The tests were repeated with flues open to look at combining the small cracks in the building envelope with large holes.

Standard methods for fan pressurization exist [3] and [4]. Both standards have recommended values for the pressure differences at which to take measurements. These pressure differences cover a range of 15 to 50 Pa for CGSB tests and 12.5 to 75 Pa for ASTM tests. Most of the time the actual pressures caused by wind and temperature difference (stack) effects on a building will be considerably less than 10 Pa. It is a fair question to ask if test results from high pressures may be extrapolated to the lower pressures that a building envelope usually experiences, because at lower flow rates the flow characteristics of the leaks may be different. This would imply that a different flow coefficient, C, and flow exponent, n, apply at the low pressures that a building experiences due to natural conditions than at the elevated pressures of a fan pressurization test.

For this study, fan pressurization tests were conducted at the Alberta Home Heating Research Facility (AHHRF) located south of Edmonton, Alberta, Canada. The houses were unoccupied and the fan pressurization test system was automated, which allowed over 5,000 fan pressurization tests to be performed. Windspeed, wind direction, and ambient temperature data were taken from meteorological towers at the test site. Pressure and flow rate measurements were taken over 15 seconds (at about 10 samples per second) and averaged for each data point. The uncertainty in the measured flows is estimated to be 0.001 m^3/s.

The indoor-outdoor pressure difference was measured using a pressure averaging manifold that had a pressure tap on each wall of the building. Offset pressures due to stack and wind effects with the fan not in operation were measured at every data point. A damper was closed over the fan opening for each offset reading because the fan opening can change the pressure distribution of the building significantly. The data shown in the following figures were chosen from tests with low windspeeds because increasing windspeed tends to increase the scatter in the measured data due to differences in the wind induced envelope pressures between the offset and measurement. For these tests, the uncertainty in the envelope pressure measurement is estimated to be 0.1 Pa.

Figure 2 shows the results of a typical test in a house with very little envelope leakage both with and without an open 15 cm diameter furnace flue with a 7.5 cm diameter orifice at the bottom (House #1 at AHHRF). The value of flow exponent (n = 0.56) with the flue open is lower than with the flue closed (n=0.73) because the flue flow exponent is about 1/2, and performing a test with the flue open will bring the value of the flow exponent for the whole building closer to 1/2. Figure 3 shows the results of a test performed in House #2 at AHHRF with a 15 cm diameter furnace flue with a 7.5 cm diameter orifice at the bottom. Curves showing the least squares fitted power law and the quadratic leakage function are also shown in Figures 2 and 3. The quadratic was matched to the least squares power law at 1 and 100 Pa to determine A and B for Equation 3. Matching at these extreme values (rather than, for example, 5 and 50 Pa or by least squares) minimizes the differences between the extrapolations of the two methods to higher and lower pressures. The results shown in Figures 2 and 3 show that the power law formulation works well for houses with an array of small cracks as well as in houses with additional large holes (in this case a furnace flue).

A significant observation to be made from the results of these tests is that the relationship between flow rate and pressure difference does not change over the range of values tested.

$C = 0.0118$, $n = 0.54$ reversed flow

The exponent, n, is the same in both cases, but the leakage coefficient C, is 13% less for backdraughting (reversed flow) most likely due to the change in flow geometry through the raincap. Using this measured value of n, together with the appropriate values of ν, z, m and ϕ, Equations 10 and 11 predict $C = 0.0127$. This shows that the theoretical power law formulations are good at estimating the flow coefficients.

Figures 4 and 5 also include a curve representing the quadratic relationship, where the flow coefficients A and B were found by calculating flow rates at 0.1 and 10 Pa and solving the two resulting equations. The tendency of this quadratic relationship to describe the flow as more laminar at low flow rates and more turbulent at higher flow rates can be seen by comparison with the reference lines indicating a slope of 1.0 (laminar flow) and a slope of 0.5 (turbulent flow). There is no clear transition from laminar to turbulent flow (like that suggested by the quadratic equation) in the measured data. This transition may have been expected because the Reynolds number has a range from approximately 600 at a flow rate of 0.001 m^3/s to over 30,000 at 0.05 m^3/s. This change in exponent is not seen because the flow is never fully developed for the whole flue and the entry and exit losses have a square root of pressure relationship. For $Re \approx 600$ the length of pipe required is 18 pipe diameters (2.7 metres) for fully developed laminar flow. Therefore approximately one half of the flue length could contain fully developed laminar flow. Similarly at higher Reynolds numbers ($Re \cong 30{,}000$) the flow in the flue is not all fully developed turbulent flow.

At flow rates less than 0.002 m^3/s there appears to be a small change in slope where the slope is increasing with decreasing flow rate, indicating that the friction factor loss term is significant and not constant with flow rate i.e. there is more laminar flow friction factor contribution. This situation occurs with flow in either direction. It should be noted that this occurs at a pressure difference of less than 0.1 Pa (which is extremely difficult to measure) and is somewhat obscured by uncertainty in the measurements, and that steady flows of this magnitude do not occur in building ventilation due to fluctuations in wind induced pressures. In addition, any mean flow generated by a 0.1 Pa pressure difference would be insignificant in an air infiltration analysis.

These results show that the power law can be applied to a single large leak over a wide range of pressures, particularly the pressures driving natural ventilation in houses.

8 Conclusions

The power law has been compared to the quadratic formulation for field and laboratory measurements of flows though building envelopes, and the theoretical backgrounds have been discussed. The power law was found to better represent the relationship between pressure and flow for buildings with small cracks only, combinations of the small building envelope cracks and large holes (a furnace flue) and laboratory measurements of furnace flues.

The following are key points developed in this paper:

- The quadratic formulation of laminar flow ($Q \propto \Delta P$) at low flows and turbulent flow ($Q \propto \Delta P^2$) at high flows is not valid for combinations of series and parallel leaks (as found in real building envelopes) and the power law is a balance between the two possible extremes of all series and all parallel leaks.
- Experimental and theoretical evidence shows that a power law function is appropriate for developing flow in cracks. Because the flow in building leaks is mostly developing flow,

[9]ASHRAE, (1989), Chapter 23 ASHRAE, Fundamentals Handbook, ASHRAE, Atlanta, Georgia.

[10] Etheridge, D.W., (1977), "Crack Flow Equations and Scale Effect", Building and Environment, Vol.12, pp. 181-189, Pergamon Press.

[11] Baker, P.H., Sharples, S., Ward, I.C., (1987), "Air Flow Through Cracks", Building and Environment, Vol.22, No.4, pp.293-304, Pergamon Press.

[12] Sherman, M.H., (1992), "A Power Law Formulation of Laminar Flow in Short Pipes", Journal of Fluids Engineering, Vol. 114, pp. 601-605, (LBL report 29414, Lawrence Berkeley Laboratory, University of California).

[13] Boussinesq, J., (1891), Mem. Pres. Acad. Sci. Paris, 23:46.

[14] Shiller, L. and Agnew, Z., (1922), Math. Mech., 2:96.

[15] Langhaar, H., (1942), "Steady Flow in the Transition Length of a Straight Tube", J. Appl. Mech. 9:A55.

[16] Kiel, D.E., Wilson, D.J., and Sherman, M.H., (1985), "Air Leakage Flow Correlations for Varying House Construction Types", ASHRAE Trans. 1985, Vol. 91, Part 2.

[17] Shapiro, A.H., Siegel, R., and Klein, S.J., (1954), "Friction Factor in the Laminar Entry Region of a Small Tube", Proc. Second U.S. National Congress of Applied Mechanics, Michigan, 1954, ASME, pp. 733-741.

[18] Jones, W.R., (1987), private correspondence with D.J. Wilson (U. of A.), M. Modera and W.J. Fisk (L.B.L.).

[19] Honma, H., (1975), "Ventilation Of Dwellings and its Disturbances", Tekniska Meddelanden No.63 1975:2 (Vol.3) of Institutionen För Uppvärmings-och Ventilation Steknik, Teknika Högskolan.

[20] Kreith, F., and Eisenstadt, R., (1957), "Pressure Drop and Flow Characteristics of Short Capillary Tubes at Low Reynolds Numbers", ASME Trans., July 1957, pp. 1070-1078.

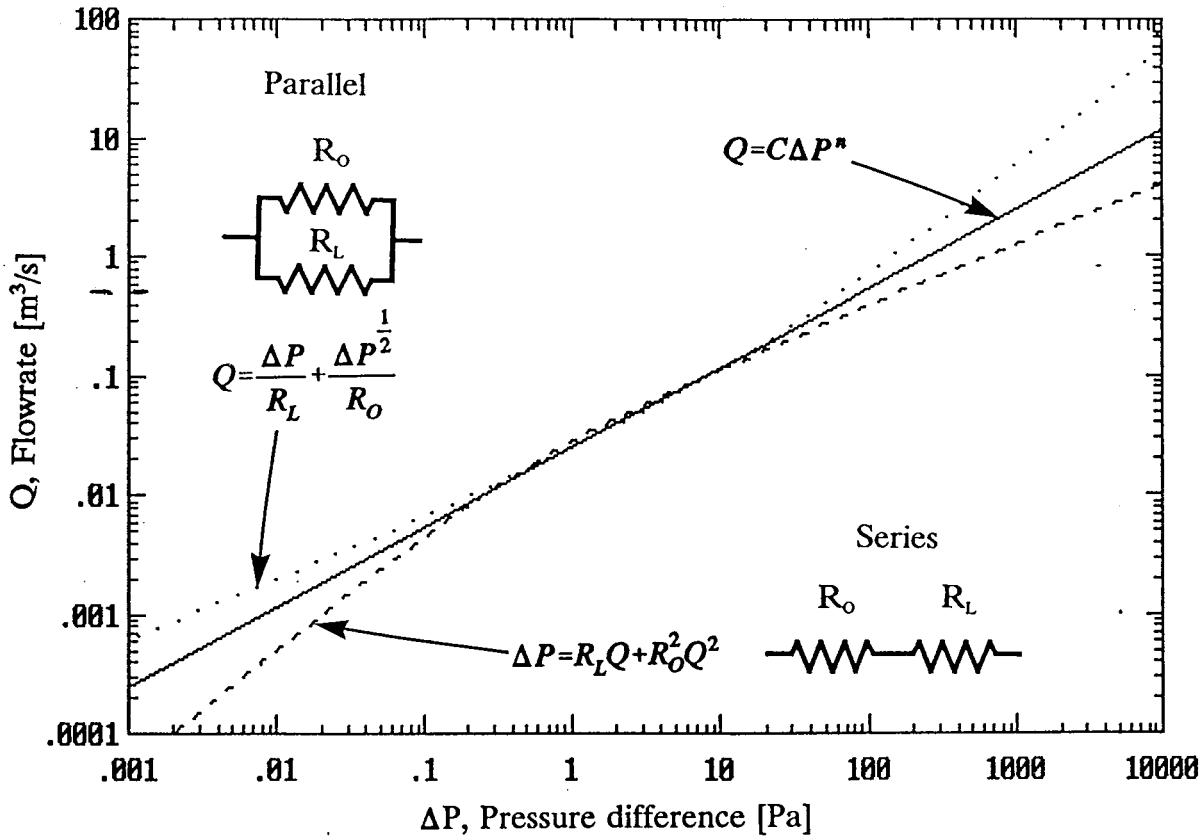

Figure 1 Illustration of series, parallel and power law crack flow model behaviour.

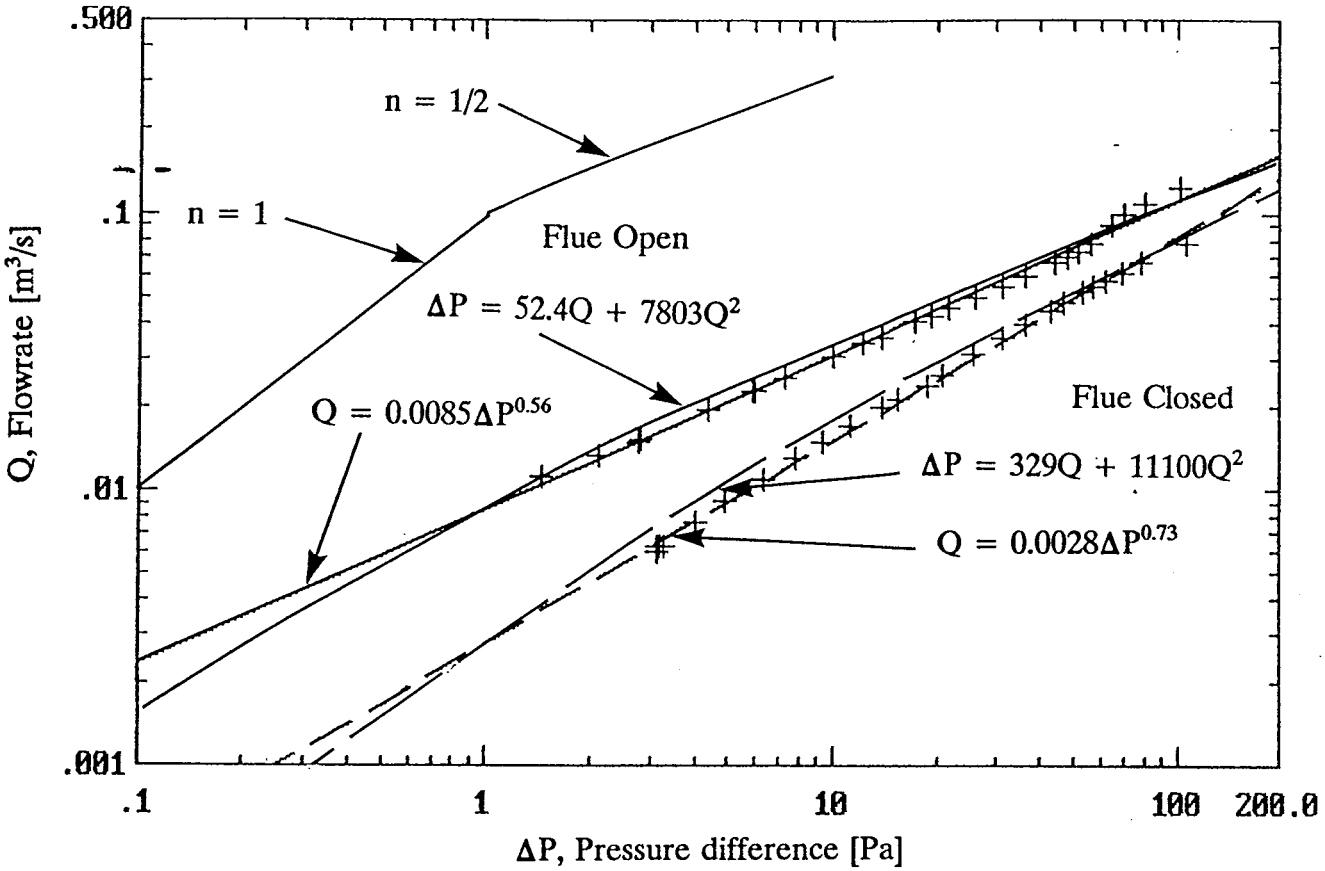

Figure 2 Blower test results for a brick walled house, with an open 15 cm I.D. flue with a 7.5 cm diameter orifice at the bottom, and with a blocked flue. Windspeed < 1.0 m/s and $\Delta T < 10^{\circ}C$.

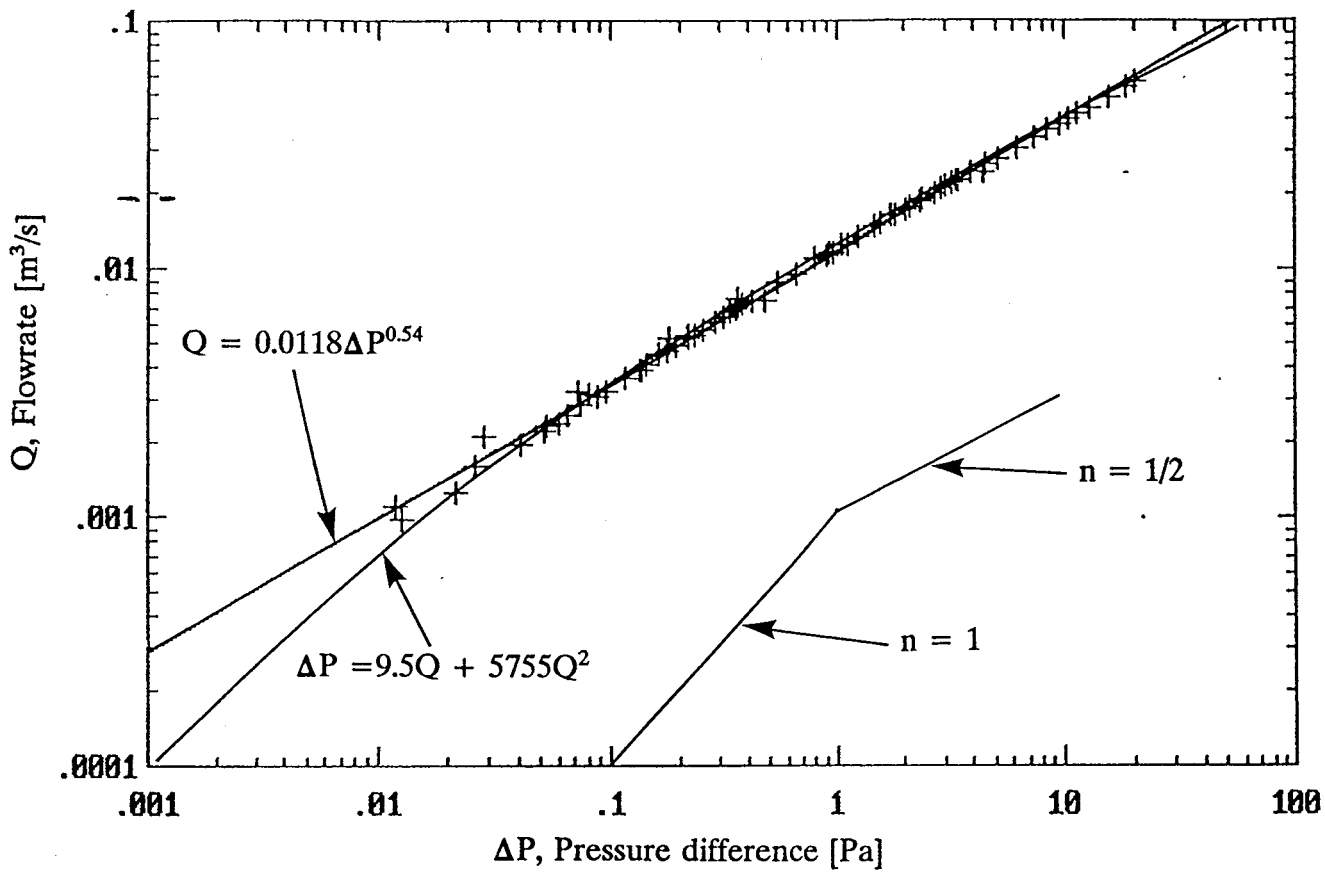

Figure 5 Pressure-flowrate relationship for 15cm diameter furnace flue with reversed flow (backdraughting)